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RAN-1901001101020001

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M.A. External (Sem.-I) Examination March - 2026

Mathematics - Complex Analysis (Paper-402)

Time: 3 Hours] **[Total Marks: 100**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book

(2) There are five questions in this question paper

(3) Answer all questions

(4) Figure to the right indicates full marks of the questions.

Seat No.:

Student's Signature

Q.1.

A. If $|z-1| = |z+1|$, find the locus of z. **07**

B. Prove that the modulus of the difference of two complex numbers is less than or equal to the sum of their moduli. **07**

C. Express $(2-i)^3$ in the form $x + iy$. **06**

OR

A. Find all values of z satisfying $z^3 = -8$. **07**

B. Show that conjugate of $(z_1 + z_2) = \bar{z}_1 + \bar{z}_2$. **07**

C. Describe geometrically $|z + 2i| \leq 3$. **06**

Q.2.

A. Prove that if $f(z) = u + iv$ is analytic, then u and v are harmonic functions. **07**

B. Find an analytic function whose imaginary part is $e^x \sin y$. **07**

C. Verify Cauchy-Riemann equations for $f(z) = z^3$ **06**

OR

A. Prove that the function $f(z) = |z|^2$ is nowhere analytic. **07**

B. If f and g are differentiable at z, prove that fg is differentiable at z. **07**

C. Find harmonic conjugate of $u - x^2y - y^3/3$. **06**

Q.3.

A. Prove Cauchy Integral Formula. **07**

B. Using Cauchy Integral Formula, evaluate $\oint |z| = 2\ z^2/(z-1)\ dz$. **07**

C. Evaluate $\int\gamma\ (z+1)\ dz$, where γ is straight line from -1 to $1 + i$. **06**

OR

A. State and prove Liouville's Theorem. **07**

B. Deduce Fundamental Theorem of Algebra. **07**

C. Evaluate $\int_{|z|=1} z\ dz$. **06**

Q.4.

A. Expand $f(z) = 1/(z-1)$ about $z = 0$. **07**

B. Define isolated singularity and classify it. **07**

C. Find residue of $f(z) = 1/z^3$ at $z = 0$. **06**

OR

A. State and prove Maximum Modulus Principle. **07**

B. Prove that an analytic function with constant modulus is constant. **07**

C. Find the order of pole of $f(z) = 1/(z-2)^4$. **06**

Q.5.

A. Find residue of $f(z) = 1/(z^2 + 1)$. **07**

B. Evaluate $\int_{|z|=3} \frac{dz}{z^2+4}$ **07**

C. Determine nature of singularity of $f(z) = e^{1/z}$ at $z = 0$. **06**

OR

A. If f has pole of order m at a, prove that residue formula holds. **07**

B. Evaluate $\oint$ $|z| = 2$ e^z/z dz. **07**

C. Show that residue at infinity equals negative sum of residues at finite poles. **06**
